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MAGNETIC FIELD INTENSITY DUE TO A LINE OF CURRENT OF ARBITRARY ORIENTATION IN CARTESIAN COORDINATES

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ABSTRACT

The **H**-field in textbooks is derived by a finite current element along the z-axis. This technical memorandum derives the **H**-field from a current element at arbitrary orientation in terms of the endpoints of the current element. This result is useful for a revision of IEMCADS as part of the EMC effort.

ADMINISTRATIVE INFORMATION

This effort was designed to improve electromagnetic (EM) analysis techniques that can be used to supplement the Intelligent Electromagnetic Analysis and Design System (IEMCADS) developed by the Electromagnetic Compatibility Branch (Code 3431).

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MAGNETIC FIELD INTENSITY DUE TO A LINE OF CURRENT OF ARBITRARY ORIENTATION IN CARTESIAN COORDINATES

Textbooks give the magnetic field intensity of a line of current in cylindrical coordinates as

$$\mathbf{H} = \frac{1}{4\pi r}(\sin\alpha_2 - \sin\alpha_1)\,\hat{a}_\phi\,, \tag{1}$$

where the line of current is along the z-axis. The angles α_2 and α_1 are given in figure 1. The quantity ρ is perpendicular distance to the z-axis. The angular unit vector in cylindrical coordinates is $\hat{a}_\phi$. However, for use in the Intelligent EMC Analysis and Design System (IEMCADS), the magnetic field intensity has more utility if expressed in cartesian coordinates and alloting for arbitrary orientations of the line of current.

Figure 2 gives the configuration of the line current and coordinates of the final result. The final result gives the magnetic field intensity in terms of the endpoints of the line current, the current, and the perpendicular distance to the axis of the line current. The derivation of this result is as follows:

The vector $\boldsymbol{\ell}$ has a direction given by

$$\boldsymbol{\ell} = (X_2 - X_1)\,\hat{a}_X + (Y_2 - Y_1)\hat{a}_Y + (Z_2 - Z_1)\hat{a}_Z\,. \tag{2}$$

The final expression must be in terms of the distance d perpendicular to the line of charge. To do this, the point is introduced so that a line drawn from the point where the field is completed or measured is perpendicular to the axis of the line of current. In figure 2 this means we introduce the point $P_3(X_3,Y_3,Z_3)$ along the axis of the line current such that the vector $\mathbf{P_3P_1}$ is perpendicular to $\mathbf{P_3P_4} = \mathbf{d}$ where 4 is the point of field computation or measurement. These vectors are given by

$$\mathbf{P_3P_4} = (X_4 - X_3)\,\hat{a}_X + (Y_4 - Y_3)\hat{a}_Y + (Z_4 - Z_3)\hat{a}_Z \tag{3}$$

$$\mathbf{P_3P_1} = (X_1 - X_3)\,\hat{a}_X + (Y_1 - Y_3)\hat{a}_Y + (Z_1 - Z_3)\hat{a}_Z\,. \tag{4}$$

The mathematical requirements for the location of point 3 are given by the dot product

$$\mathbf{P_3P_1} \bullet \mathbf{P_3P_4} = 0 \tag{5}$$

and the equation of the straight line defining this axis is

$$\frac{X_3 - X_1}{a} = \frac{Y_3 - Y_1}{b} = \frac{Z_3 - Z_1}{c}\,. \tag{6}$$

Equation (6) can be written as two equations

$$X_3 = \frac{a}{c}(Z_3 - Z_1) + X_1 \tag{7}$$

and

$$Y_3 = \frac{b}{c}\,(Z_3 - Z_1) + Y_1\,. \tag{8}$$

Equation (5) expressed explicitly in cartesian coordinates is

$$(X_4 - X_3)(X_1 - X_3) + (Y_4 - Y_3)(Y_1 - Y_3) + (Z_4 - Z_3)(Z_1 - Z_3) = 0\,. \tag{9}$$

Substituting 7 and 8 into 9 yields

$$\left[X_4 - \left\{\frac{a}{c}(Z_3 - Z_1) + X_1\right\}\right]\left[-\frac{a}{c}(Z_3 - Z_1)\right]$$

$$+\left[X_4 - \left\{\frac{b}{c}(Z_3 - Z_1) + Y_1\right\}\right]\left[-\frac{b}{c}(Z_3 - Z_1)\right]$$

$$+ [Z_4 - Z_3]\,[Z_1 - Z_3] = 0\,. \tag{10}$$

This equation is quadratic in the unknown Z_3 and can be solved by the quadratic equation since X_1, Y_1, Z, X_1. Y_1, Z_1, and X_4, Y4, Z_4 are considered known quantities. Once equation (10) is solved for Z_3, X_3, and Y_3 are easily obtained from equations (7) and (8).

The solution of X_3, Y_3 and Z_3 combined with the known solution of X_4, Y_4, and Z_4 will yield the perpendicular distance d from the point of field measurement to the axis of the line current. The quantity d is given by

$$d = \sqrt{(X_3 - X_4)^2 + (Y_3 - Y_4)^2 + (Z_3 - Z_4)^2}\,. \tag{11}$$

The magnetic field intensity for the arbitrary orientation in cartesian coordinates is finally

$$\mathbf{H} = \frac{I}{4\pi d}\,[\sin\beta_2 \hat{a}_{\perp 2} - \sin\beta_1 \hat{a}_{\perp 1}] \tag{12}$$

where

$$\sin\beta_2 = \frac{d_{32}}{d_{24}} \tag{13}$$

and

$$\sin\beta_1 = \frac{d_{31}}{d_{14}}\,. \tag{14}$$

The distances d_{32}, d_{24}, d_{31}, and d_{14} are given by

$$d_{32} = \sqrt{(X_3 - X_2)^2 + (Y_3 - Y_2)^2 + (Z_3 - Z_2)^2} \tag{15}$$

$$d_{31} = \sqrt{(X_3 - X_1)^2 + (Y_3 - Y_1)^2 + (Z_3 - Z_1)^2} \tag{16}$$

$$d_{24} = \sqrt{(X_1 - X_4)^2 + (Y_1 - Y_4)^2 + (Z_1 - Z_4)^2} \tag{17}$$

$$d_{14} = \sqrt{(X_1 - X_4)^2 + (Y_1 - Y_4)^2 + (Z_1 - Z_4)^2} \tag{18}$$

The unit vectors $\hat{a}_\perp$ are given by

$$\hat{a}_{\perp 2} = \frac{\mathbf{P_3P_2} \times \mathbf{P_3P_4}}{|\mathbf{P_3P_2} \times \mathbf{P_3P_4}|} \tag{19}$$

and

$$\hat{a}_{\perp 1} = \frac{\mathbf{P_3P_1} \times \mathbf{P_3P_4}}{|\mathbf{P_3P_1} \times \mathbf{P_3P_4}|} . \tag{20}$$

These unit vectors can explicitly be written in terms of the end points of the current element and the now known coordinate X_3, Y_3, and Z_3.

To do this we express $\mathbf{P_3P_1}$, $\mathbf{P_3P_4}$, and $\mathbf{P_3P_2}$in known cartesian coordinates

$$\mathbf{P_3P_1} = (X_3 - X_1)\hat{a}_X + (Y_3 - Y_1)\hat{a}_Y + (Z_3 - Z_1)\hat{a}_Z \tag{21}$$

$$\mathbf{P_3P_4} = (X_3 - X_4)\hat{a}_X + (Y_3 - Y_4)\hat{a}_Y + (Z_3 - Z_4)\hat{a}_Z \tag{22}$$

$$\mathbf{P_3P_2} = (X_3 - X_2)\hat{a}_X + (Y_3 - Y_2)\hat{a}_Y + (Z_3 - Z_2)\hat{a}_Z . \tag{23}$$

Taking the cross products one merely takes the determinant of the following matrices

$$\mathbf{P_3P_1 \times P_3P4} = \begin{vmatrix} \hat{a}_X & \hat{a}_Y & \hat{a}_Z \\ (X_3 - X_1) & (Y_3 - Y_1) & (Z_3 - Z_1) \\ (X_3 - X_4) & (Y_3 - Y_4) & (Z_3 - Z_4) \end{vmatrix} \tag{24}$$

Taking the determinant of equation (24) yields (25)

$$= \hat{a}_X[(Y_3 - Y_1)(Z_3 - Z_4) - (Y_3 - Y_4)(Z_3 - Z_1)]$$

$$+ \hat{a}_Y[(Z_3 - Z_1)(X_3 - X_4) - (X_3 - X_1)(Z_3 - Z_4)]$$

$$+ \hat{a}_Z[(X_3 - X_1)(Y_3 - Y_4) - (Y_3 - Y_1)(X_3 - X_4)] \tag{25}$$

$$\mathbf{P_3P_2 \times P_3P4} = \begin{vmatrix} \hat{a}_X & \hat{a}_Y & \hat{a}_Z \\ (X_3 - X_2) & (Y_3 - Y_2) & (Z_3 - Z_2) \\ (X_3 - X_4) & (Y_3 - Y_4) & (Z_3 - Z_4) \end{vmatrix} \tag{26}$$

Taking the determinant of equation (26) yields

$$= \hat{a}_X[(Y_3 - Y_2)(Z_3 - Z_4) - (Z_3 - Z_2)(Y_3 - Y_4)]$$

$$+ \hat{a}_Y[(Z_3 - Z_2)(X_3 - X_4) - (X_3 - X_2)(Z_3 - Z_4)]$$

$$+ \hat{a}_Z[(X_3 - X_2)(Y_3 - Y_4) - (X_3 - X_4)(Z_3 - Z_2)] \tag{27}$$

Taking the absolute values, we obtain

$$\left|\mathbf{P_3P_1 \ x \ P_3P_4}\right|$$

$$= ([(Y_3 - Y_1)(Z_3 - Z_4) - (Y_3 - Y_4)(Z_3 - Z_1)]^2$$

$$+ [(Z_3 - Z_1)(X_3 - X_4) - (X_3 - X_1)(Z_3 - Z_4)]^2$$

$$+[(X_3 - X_1)(Y_3 - Y_4) - (Y_3 - Y_1)(X_3 - X_4]^2)^{1/2} \quad (28)$$

$$\left|\mathbf{P_3P_1 \ x \ P_3P_4}\right|$$

$$= ([(Y_3 - Y_2)(Z_3 - Z_4) - (Z_3 - Z_2)(Y_3 - Y_1)]^2$$

$$+ [(Z_3 - Z_2)(X_3 - X_4) - (X_3 - X_2)(Z_3 - Z_4)]^2$$

$$+[(X_3 - X_2)(Y_3 - Y_4) - (X_3 - X_4)(Y_3 - Y_2]^2)^{1/2} \quad (29)$$

The final form for the **H**-field at arbitrary orientations is

$$\mathbf{H} = \frac{I}{4\pi d}[\sin\beta_2 \hat{a}_{\perp 2} - \sin\beta_1 \hat{a}_{\perp 1}] \quad (30)$$

where d is given by equation (11), β_2 and β_1 are given in equations (13) through (18), and $\hat{a}_{\perp}$is given by equations (19) through (29).

CONCLUSIONS

The final expression for the **H**-fields is given by equation (30) and all the appropriate substitutions. The final result gives the field at some arbitrary point in terms of the endpoints of the current element, the current, and the perpendicular distance to the axis of the current element. This result differs from the current elements presently in IEMCADS in that it gives the resulting field with the current element at arbitrary orientations. IEMCADS is presently limited at giving fields for current elements fixed along the z-axis and consequently is of limited value. This result will be incorporated into IEMCADS to give greater flexibility in its EMI predictions.

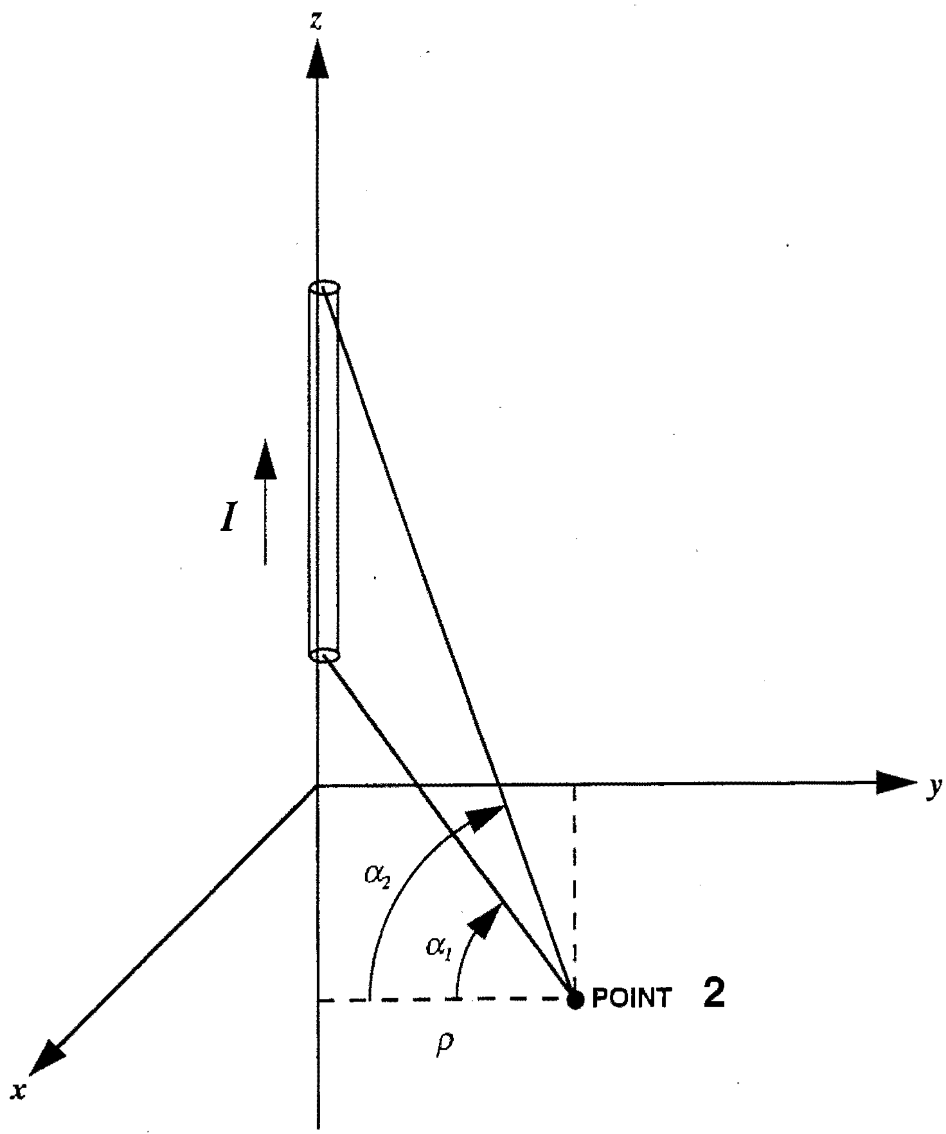

Figure 1. The **H**-field caused by a finite current element on the z-axis. The **H**-field at Point 2 is given by equation (1).

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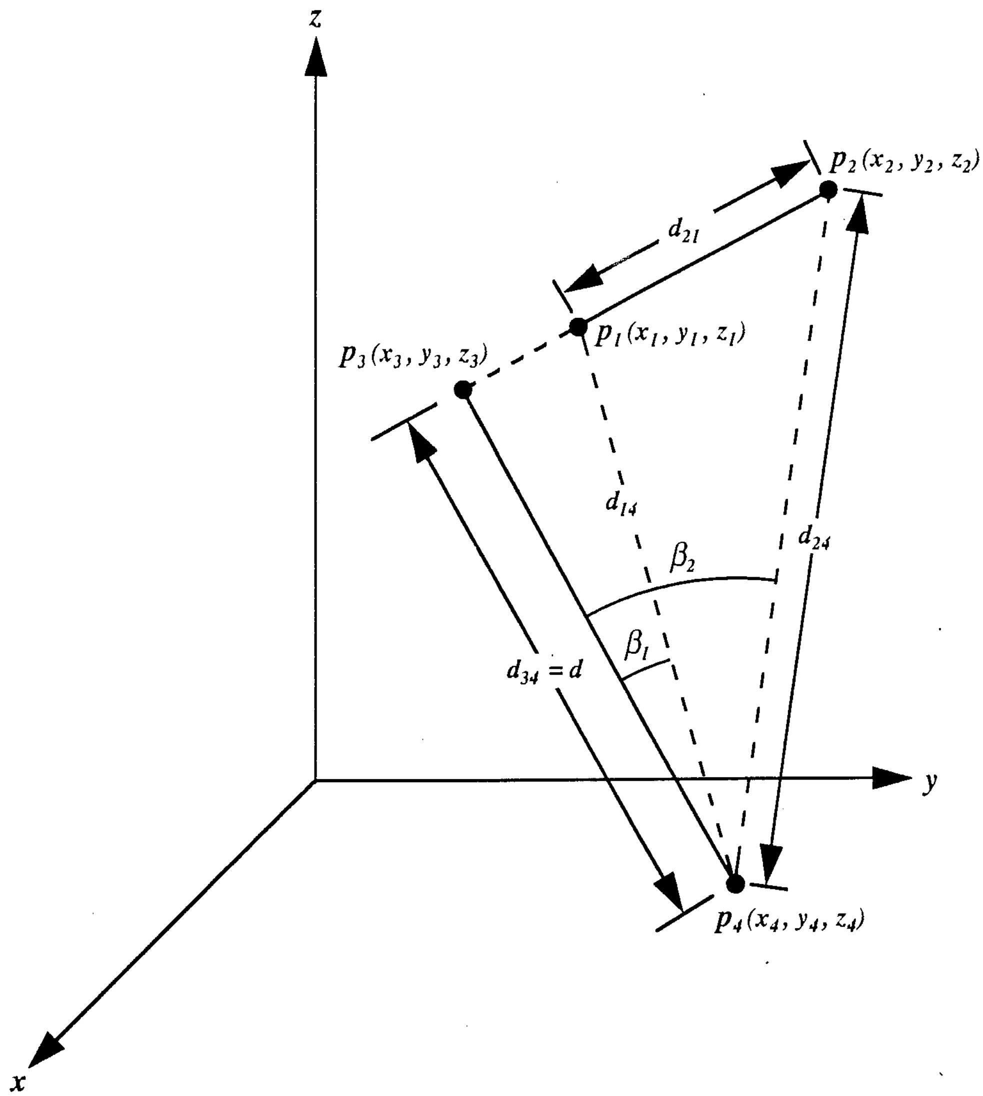

Figure 2. The **H**-field at the point P_4 in this arbitrary orientation is given by equation (31).

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